Bond Future Option

FinPricing
A bond future option is an option contract that gives the holder the right but not the obligation to buy or sell a bond future at a predetermined price.

Bond future options offer significant advantages for reducing costs, enhancing returns and managing risk.

Bond future options could be European style or American style.

The writer/seller receives a premium from the buyer for undertaking this obligation.

Options are leveraged instruments that allow the owner to control a large amount of the underlying asset with a smaller amount of money.
Bond future options are also exchange traded derivatives on treasury instruments.

Bond future options provide market participants with the ability to adjust their interest rate exposures.

A bond future option is also a good tool for hedging, income enhancement, duration adjustments, interest rate speculation and spread trading.

Investors use bond future options to hedge an existing portfolio against adverse interest rate movements or enhance the long-term performance.

Arbitrageurs profit from the price difference between the spot bonds and the bond futures.

Speculators use bond future option in the hope of making a profit on short-term movements in prices.
The present value of a call bond future option is represented as:

\[ PV(0) = N[F_T \Phi(d_1) - K \Phi(d_2)]D_T \]

The present value of a put bond future option is represented as:

\[ PV(0) = N[K \Phi(-d_2) - F_T \Phi(-d_1)]D_T \]

where

- \( d_{1,2} = \frac{\ln(F_T/K) \pm \sigma^2 T/2}{\sigma \sqrt{T}} \)
- \( F_T = [(P - C_\Sigma) \exp(r_T T) - A] / CF \) the forward clean price of the delivered bond (CTD) at time 0.
- \( C_\Sigma = \sum_{t_i \leq T} C \exp(-r_i t_i) \) the summed present value of all coupons of the underlying bond between 0 and T.
- \( K \) the strike.
Valuation (Cont.)

- $N$ the notional.
- $T$ the option maturity date.
- $D_T$ the discount factor.
- $CF$ the conversion factor for a bond to deliver in a bond futures contract.
- $A$ the accrual interest before $T$.
- $P$ the bond dirty price at 0.
- $r_T$ the continuously compounded interest rate between $t$ and $T$.
- $\sigma = \frac{\alpha Dy \sigma_y}{CF}$ the volatility of forward bond price.
Valuation (Cont.)

- $\sigma_y$ the forward yield volatility of the CTD bond of the underlying futures. We use the swaption volatility
- $\alpha$ the implied volatility scaling factor.
- $y$ the forward yield that can be solved by
  \[
  P - C_\Sigma = \sum_{T \leq t_i \leq T_B} C e^{-y t_i}
  \]
- $T_B$ the maturity of the underlying CTD bond
- $D = \frac{\sum_{T \leq t_i \leq T_B} t_i C e^{-y t_i}}{\sum_{T \leq t_i \leq T_B} C e^{-y t_i}}$ the forward modified duration of the CTD bond of the underlying futures
Valuation (American Style)

- We use the Cox-Ross-Rubinstein (CRR) binomial tree to price American bond future option.
- Build forward bond price tree.

\[ F_0 = \left[ (P - C_\Sigma) \exp(r_T T) - A \right] / CF \]

\[ F_j^u = F_j e^{\sigma \sqrt{\Delta t}} \text{ with probability } p = \frac{1 - e^{-\sigma \sqrt{\Delta t}}}{e^{\sigma \sqrt{\Delta t}} - e^{-\sigma \sqrt{\Delta t}}} \text{ where } \Delta t = T / m \]

\[ F_j^u = F_j e^{-\sigma \sqrt{\Delta t}} \text{ with probability } 1 - p \text{ where } j = 1, \ldots, m \]

\[ \sigma = \alpha Dy \sigma_y / CF \text{ is the volatility described above} \]

- After constructing the tree, valuation is performed backward until the valuation date. The option value at node 0 is the present value of the bond future option.
Notes

▪ First compute the CTD forward bond price first.

▪ Then determine the volatility of the forward bond price.

▪ After that, call Black formula for pricing European bond future options.

▪ Or build binomial tree to value American bond future options.
## Bond Future Option

### Example

<table>
<thead>
<tr>
<th>Option Specification</th>
<th>Future Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Buy Sell</strong></td>
<td><strong>Buy</strong></td>
</tr>
<tr>
<td><strong>Call Put</strong></td>
<td><strong>Call</strong></td>
</tr>
<tr>
<td><strong>Currency</strong></td>
<td><strong>USD</strong></td>
</tr>
<tr>
<td><strong>Option Maturity Date</strong></td>
<td><strong>5/26/2017</strong></td>
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<tr>
<td><strong>Option Expiry Date</strong></td>
<td><strong>5/26/2017</strong></td>
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<tr>
<td><strong>Strike</strong></td>
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<td><strong>Option Ticker</strong></td>
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<tr>
<td><strong>Settlement Amount</strong></td>
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<td><strong>Settlement Date</strong></td>
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<tr>
<td><strong>Trade Date</strong></td>
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<td><strong>Future Maturity Date</strong></td>
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<tr>
<td><strong>Underlying Bond Type</strong></td>
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<tr>
<td><strong>Underlying Bond Coupon</strong></td>
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<tr>
<td><strong>Underlying Bond Maturity Date</strong></td>
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</tr>
</tbody>
</table>
Reference:

https://finpricing.com/lib/EqRainbow.html