Historical Value at Risk
Value at Risk Definition

- The maximum likely loss on a portfolio for a given probability defined as $x\%$ confidence level over $N$ days
- $\Pr(\text{Loss} > \text{VaR}(x\%)) < 1 - x\%$
Historical VaR

Value at Risk Roles

- Risk management
- Risk control
- Financial reporting
- Regulatory and economic capital
Historical VaR

Value at Risk Pros & Cons

◆ Pros
  ◆ Regulatory measurement for market risk
  ◆ Objective assessment
  ◆ Intuition and clear interpretation
  ◆ Consistent and flexible measurement

◆ Cons
  ◆ Doesn’t measure risk beyond the confidence level: tail risk
  ◆ Non sub-additive
Three Value at Risk Approaches

- Parametric Value at Risk
- Historical Value at Risk
- Monte Carlo Value at Risk
Historical VaR

Historical Value at Risk

Assumption

The past is a good indicator of the near-future or history repeats itself

Pros

- Simple and intuitive
- Easy back and stress test
- No distribution assumption
- No calibration

Cons

- Poor accuracy for higher confidence level and tail risk
- Difficult for long horizons
- Limited scenario
Historical Value at Risk Methodology and Implementation

- Obtain one year historical value time series of all market factors, such as a stock price time series is $\bar{x}_1 \ldots \bar{x}_{251}$
- Assuming today’s value is $x_0$, generate 250 historical scenarios. The i-th is $x_i = (\bar{x}_i/\bar{x}_{i-1} - 1)x_0$
- Compute base PV at today t as $P(x_o)$
- Compute 250 scenario PVs: $P(x_i)$
- Compute 250 scenario P&L: $P(x_i) - P(x_0)$
- Sort 250 scenario P&L. The Value at Risk is the average between 2nd and 3rd lowest (negative) numbers
Historical VaR

Value at Risk Scaling

- Normally firms compute 1-day 99% Value at Risk
- Regulators require 10-day 99% Value at Risk
- Under IID assumption, 10-day Value at Risk = $\sqrt{10} \times VaR_{1-day}$
The only way to verify a Value at Risk system is to backtest.

At a certain day, compute hypothetic P&L. If \((\text{hypothetic P&L} > \text{VaR})\) breach, otherwise, ok.

Hypothetic P&L is computed by holding valuation date and portfolio unchanged.

In one year period,

- If number of breaches is 0-4, the Value at Risk system is in Green zone.
- If number of breaches is 5-9, the Value at Risk system is in Yellow zone.
- If number of breaches is 10 or more, the Value at Risk system is in Red zone.
Reference:

https://finpricing.com/lib/EqConvertible.html